Mark Scheme (Results)

January 2021

Pearson Edexcel International A Level
in Statistics S2 (WST02/01)

Question Number	Scheme		Marks
1(a)	B(30, 0.05)		B1
			(1)
(b)	The **probability**(oe) of an oyster surviving/not surviving is **constant**		B1
	The survival of each oyster is **independent** of the others		(1)
(c)(i)	${}^{30}C_{24}(0.05)^6(0.95)^{24}$ oe		M1
	$= 0.002708\ldots$	awrt 0.0027	A1
(ii)	$P(Y \geqslant 3) = 1 - P(Y \leqslant 2)$ from $Y \sim B(30, 0.05)$ or $P(X \leqslant 27)$ from $X \sim B(30, 0.95)$		M1
	$= 1 - 0.8122$		
	$= 0.1878$	awrt 0.188	A1
			(4)
(d)	$A \sim Po(10)$		B1
	$P(A \geqslant n) > 0.8$		
	$P(A \leqslant n-1) < 0.2$ or $P(A \leqslant 6) = 0.1301$....awrt 0.13 or $P(A \geqslant 7) = 0.8699$....awrt 0.87		M1
	$n = 7$		A1cao
			(3)
(e)	$H_0 : p = 0.05$, $H_1 : p > 0.05$		B1
	Using $C \sim B(25, 0.05)$ and $P(C \geqslant 4)$ / Using $D \sim B(25, 0.95)$ and $P(D \leqslant 21)$		M1
	$P(C \geqslant 4) = 0.0341$ / CR $C \geqslant 4$ / $P(D \leqslant 21) = 0.0341$ / CR $D \leqslant 21$		A1
	Evidence to reject H_0, in the CR, significant		dM1
	There is evidence that the proportion of **oysters** not surviving has **increased**(oe)/**Jim's belief** is supported.		A1cso
			(5)
			Total 14

Notes		
(a)	**B1**	Must include B(inomial), $n = 25$ and $p = 0.05$. Do not allow $p = 0.95$ in part (a)
(b)	**B1**	For either correct assumption in context. Ignore extraneous non-contradicting comments.
(c)(i)	**M1**	allow ${}^{30}C_6$ oe or $P(X \leq 6) - P(X \leq 5)$ with one correct probability
	A1	awrt 0.0027 (correct answer scores 2 out of 2)
(ii)	**M1**	Writing/using $1 - P(Y \leqslant 2)$ with B(30, 0.05) or writing/using $P(X \leqslant 27)$ with B(30, 0.95)
	A1	awrt 0.188 (correct answer scores 2 out of 2)
(d)	**B1**	Writing or using Po(10) (sight of 0.1301 or 0.8699 can imply this mark)
	M1	Allow $P(A < n) < 0.2$ or $P(A < 7) =$ awrt 0.13 or $P(A > 6) =$ awrt 0.87
	A1cao	$n = 7$ which must come from use of Po(10) or N(10, 9.5)
	Note:	Use of normal approx. with $\mu = 10$ and $\sigma^2 = 9.5$ leading to $n < 7.4\ldots$can score M1 Exact binomial gives $P(A \leqslant 6) = 0.14$ / $P(A \geqslant 7) = 0.86$ scores B0M0A0
(e)	**B1**	Both hypotheses correct (allow use of p or π). Allow $H_0 : p = 0.95$, $H_1 : p < 0.95$
	M1	Using B(25, 0.05) and writing/using $P(C \geqslant 4)$ or if CR given $P(C \geqslant 3)$ using B(25, 0.95) and writing/using $P(D \leqslant 21)$ or if CR given $P(D \leqslant 20)$
	A1	Correct probability to 3sf (must not go on and give incorrect CR) or correct CR (ignore upper tail)
	dM1	(dep on 1st M1) A correct non-contextual statement (do not allow contradicting non-contextual comments) which is consistent with their prob and 0.05 (If not stated, may be implied by A1)
	A1cso	All previous marks must be awarded. Correct contextual conclusion with bold words (oe)
SC:	**2-tail**	Use of two-tailed test can score max: B1M1A1M1A0, but must **not reject** H_0 for 2nd M1

Question Number	Scheme		Marks
2(a)	$1 - F(3.5) = 1 - 0.97127\ldots$		M1
	$= 0.028727\ldots$	awrt 0.0287	A1
			(2)
(b)	$W \sim B(30, "0.0287")$		M1
	$1 - P(W \leq 1) = 1 - \left((1 - "0.0287")^{30} + {}^{30}C_1 ("0.0287")^1 (1 - "0.0287")^{29} \right)$ oe		M1
	$= 1 - 0.78748\ldots = 0.2125\ldots$	awrt 0.213 to awrt 0.216	A1
			(3)
(c)	$\frac{dF(w)}{dw} = \frac{1}{3}\left(1 - \frac{w^3}{64}\right)$		M1
	$E(W^2) = \int_0^4 \frac{1}{3}\left(w^2 - \frac{w^5}{64}\right) dw = \frac{1}{3}\left[\frac{w^3}{3} - \frac{w^6}{384}\right]_0^4$		dM1
	$= \frac{32}{9}$		A1
	$Var(W) = \frac{32}{9} - 1.6^2$		M1
	$= \frac{224}{225}$		A1
			(5)
			Total 10

Notes		
(a)	**M1**	For writing or using $1 - F(3.5)$ Implied by correct answer
	A1	awrt 0.0287
(b)	**M1**	For writing or using $B(30, "0.0287")$ allow $n("\text{their } 0.0287")^1 (1 - "\text{their } 0.0287")^{29}$ ignore any number for n (allow their p to 2sf)
	M1	For $1 - \left((1 - "0.0287")^{30} + {}^{30}C_1 ("0.0287")^1 (1 - "0.0287")^{29} \right)$ Allow ${}^{30}C_{29}$ in any form
	A1	allow answer in the range awrt 0.213 to awrt 0.216
(c)	**M1**	Differentiating F(w) at least one term correct
	dM1	(Dep on previous M1). Attempting to integrate expanded $w^2 f(w)$. At least one $w^n \rightarrow w^{n+1}$ Ignore limits for this M mark.
	A1	awrt 3.56 must come from correct algebraic integration (may be embedded)
	M1	Use of correct formula with values substituted. Must see the subtraction of 1.6^2
	A1	Dependent upon 2nd M1 awrt 0.996 (A correct answer with no algebraic integration seen may score M1M0A0M1A0)

Question Number	Scheme		Marks
3(a)	$P(X \neq 4) = 1 - P(X = 4)$ oe $\left(= 1 - \frac{e^{-7}7^4}{4!} \quad \text{or} \quad 1 - (0.1730 - 0.0818)\right)$		M1
	$= 0.90877\ldots$	awrt 0.909	A1
			(2)
(b)	$P(Y = 1) = (1 - "0.90877...")("0.90877...")^4 \times {}^5C_1$		M1M1
	$= 0.311\ldots$		A1
			(3)
(c)(i)	$\lambda = 0.07n$		B1
	$A \sim N(0.07n, 0.07n)$		M1
	$\frac{3.5 - "0.07n"}{\sqrt{"0.07n"}}$		M1
	$\frac{3.5 - 0.07n}{\sqrt{0.07n}} = -1.55$ or $"0.07n" - \left(1.55\sqrt{0.07}\right)\sqrt{n} - 3.5 = 0$		B1
	$n - \left(\frac{1.55}{0.07}\sqrt{0.07}\right)\sqrt{n} - \frac{3.5}{0.07} = 0 \Rightarrow n - 1.55\sqrt{\frac{n}{0.07}} - 50 = 0$		A1cso
			(5)
(ii)	$\sqrt{n} = \frac{\frac{1.55}{\sqrt{0.07}} \pm \sqrt{\left(\frac{1.55}{\sqrt{0.07}}\right)^2 + 4 \times 50}}{2}$ = awrt – 4.72… or awrt 10.6… ($4\sqrt{7}$)		M1
	$n = 112$		A1cao
			(2)
(d)	$H_0 : \lambda = 7 \quad H_1 : \lambda > 7$		B1
	$P(X \geq 15) = 1 - P(X \leq 14)$ $P(X \geq 14) = 0.0128$		M1
	$= 1 - 0.9943$ $P(X \geq 15) = 0.0057$		
	$= 0.0057$ CR $X \geqslant 15$		A1
	Reject H_0, in the CR, Significant		dM1
	There is evidence that the number of water **fleas** per 100 ml of the pond water has **increased**		A1
			(5)
			Total 17

Notes		
(a)	**M1**	For $1 - P(X = 4)$ or $1 - P(X \leq 4) + P(X \leq 3)$ oe
(b)	**M1**	$(1 - "\text{their } 0.909")^4 ("\text{their } 0.909")$ or $(1 - "\text{their } 0.909")("\text{their } 0.909")^4$ allow their values to 2s.f.
	M1	$P(Y = 1) = (1 - "\text{their } 0.909")("\text{their } 0.909")^4 \times {}^5C_1$ allow their values to 2s.f.
	A1	awrt 0.312 or awrt 0.311
(c)(i)	**B1**	Writing or using mean as $0.07n$
	M1	Normal with the mean = variance which must be in terms of n (may be implied by correct standardisation).
	M1	Standardising with their mean and their $\sqrt{\text{var}}$. If not stated they must be correct. Allow 2.5, 3, 3.5,4, 4.5 (A **correct** standardisation implies B1M1M1)
	B1	Their standardisation = ± 1.55
	A1cso	Must come from compatible signs in standardisation. Need at least one step between standardisation indicating division by 0.07 and correct equation.
(ii)	**M1**	Correct method to solve **given** quadratic or sight of awrt –4.72 or awrt 10.6
	A1cao	112 only (must reject 2nd answer if found) (an answer of 112 only scores M1A1)
(d)	**B1**	Both hypotheses correct in terms of λ or μ [using p scores B0]
	M1	For $1 - P(X \leq 14)$ **or** for CR: one of $P(X \geq 14) = 0.0128$ or $P(X \geq 15) = 0.0057$
	A1	awrt 0.0057 or correct CR allow $X > 14$
	dM1	(dep on 1st M1) A correct non-contextual statement (do not allow contradicting non-contextual comments) which is consistent with their prob and 0.01. (If not stated, may be implied by A1)
	A1	All previous marks must be awarded. Correct context. conclusion with increase(oe) and fleas

Question Number	Scheme		Marks
4(a)	$\int_0^a k(a-x)^2\,\mathrm{d}x = \left[k\left(a^2x - ax^2 + \frac{x^3}{3}\right)\right]_0^a$ or $\left[\frac{-k(a-x)^3}{3}\right]_0^a$		M1 A1
	$k\left(a^3 - a^3 + \frac{a^3}{3}\right) = 1$ or $\frac{ka^3}{3} = 1$ $\Rightarrow ka^3 = 3$		A1 cso
			(3)
(b)	$\int_0^a kx(a-x)^2\,\mathrm{d}x = \left[k\left(\frac{a^2x^2}{2} - \frac{2ax^3}{3} + \frac{x^4}{4}\right)\right]_0^a$ or $\left[\frac{-kx(a-x)^3}{3} + \frac{k(a-x)^4}{12}\right]_0^a$		M1A1
	$k\left(\frac{a^2a^2}{2} - \frac{2aa^3}{3} + \frac{a^4}{4}\right) = 1.5$ or $\left[\frac{ka(a)^3}{3} - \frac{k(a)^4}{12}\right]_0^a = 1.5$ or $ka^4 = 18$ oe		dM1
	$\frac{ka^4}{ka^3} = 6$ or $\frac{18}{3} = 6$ $[\therefore a = 6]$		A1cso
			(4)
(c)	$F(x) = \frac{1}{72}\left(36x - 6x^2 + \frac{x^3}{3}\right)$	$\frac{1}{72}\left(36x - 6x^2 + \frac{x^3}{3}\right) = 0.5$ oe	M1
	F(1.15)(= 0.47…) and F(1.25) (= 0.5038…)	1.2377…	M1
	F(1.15) = awrt 0.47, F(1.25) = awrt 0.504 (0.47(18…) < 0.5 < 0.503(8…)) therefore the median is **1.2** to 1 decimal place.	therefore the median is **1.2** to 1 decimal place.	A1
			(3)
			Total 10

Notes		
(a)	**M1**	Integrating f(x) at least 1 term correct. For M1 allow $\frac{\pm k(a-x)^3}{3}$
	A1	Correct integration (ignore limits)
	A1cso	Substitute limits and equating to 1 to form one expression in terms of k and a leading to $ka^3 = 3$
(b)	**M1**	Indicating that they are integrating $xf(x)$ with an attempt at integrating $x^n \rightarrow x^{n+1}$
	A1	Correct integration
	dM1	(dep on previous M1). Substitute limits and equating to 1.5 to form a 2nd expression in terms of k and a
	A1cso	Correct method shown to solve their 2 equations to eliminate k and show a=6
(c)	**M1**	Finding correct F(x). Allow $F(x) = 1 - \frac{(6-x)^3}{216}$ but $F(x) = \frac{(6-x)^3}{216}$ is M0 Allow in terms of k for this mark
	M1	For attempting their F(1.15) and their F(1.25) or a suitable tighter interval **or** for 'solving' cubic leading to a value awrt 1.24
	A1	Both correct values and correct conclusion (allow x = 1.2) **or** awrt 1.24 and correct conclusion (allow x = 1.2). Allow change of sign argument if they have subtracted 0.5 (i.e. – 0.028… < 0 < 0.0038…).

Question Number	Scheme	Marks
5(a)	U[0, 3]	M1
	$\frac{3-1.8}{3} = 0.4$	A1
		(2)
(b)	$X^2 = W^2 + (3-W)^2$	M1
	$X^2 = W^2 + 9 + W^2 - 6W \quad \Rightarrow \quad X^2 = 2W^2 - 6W + 9$	A1
		(2)
(c)	E(W) = 1.5	B1
	Var(W) = $\frac{9}{12} = \frac{3}{4}$	B1
	E(W^2)= "$\frac{3}{4}$" + "1.5"^{2}	M1
	$E(W^2) = 3$	A1
	So $E(X^2) = 2\times"3" - 6\times"1.5" + 9 = 6$	M1A1
		(6)
(d)	$P(X^2 > 5) = P\left(2W^2 - 6W + 4 > 0\right)$	M1
	$= P\left((2W-2)(W-2) > 0\right)$	M1
	$= P(W > 2) + P(W < 1)$	dM1
	$= \frac{2}{3}$ oe	A1
		(4)
		Total 14

Notes

(a)	**M1**	Writing or using the correct distribution Allow: $\frac{1.8}{3}$ for M1A0	
	A1	0.4 oe	
(b)	**M1**	Using Pythagoras to find the length Note: $X^2 = W^2 + (W-3)^2$ scores M1A0	
	A1	Brackets multiplied seen leading to $X^2 = 2W^2 - 6W + 9$ with no incorrect working	
(c)	**B1**	1.5	
	B1	Var(W) = 0.75	Using integration: $E(W^2) = \int_0^3 \frac{1}{3} w^2 \, dw$ (ignore limits)
	M1	Writing or using $E(W^2) = Var(W) + [E(W)]^2$	$\left[\frac{1}{9} w^3\right]_0^3$ (correct integration with correct limits)
	A1	3	
	M1	Use of $E(X^2) = 2E(W^2) - 6E(W) + 9$ with their values.	
	A1	6 An answer of 6 from correct working implies all previous marks.	
(d)	**M1**	For realising they need to find the probability of $2W^2 - 6W + 4 > 0$ (condone =)	
	M1	Solving their 3-term quadratic (W = 1 and W = 2 implies 1st two M marks)	
	dM1	(dep on 2nd M1) Realising they need to add the 2 outer areas	
	A1	awrt 0.667	

Question Number	Scheme	Marks
6(a)	Taking a random sample is quicker/cheaper/easier (compared to asking all of the youth club members).	B1
		(1)
(b)	A list/register/database of all the youth club members	B1
		(1)
(c)	The members	B1
		(1)
(d)	$p^2 = \frac{25}{64}$	M1
	$p = \frac{5}{8}$	A1
	$"\frac{5}{8}" + q + r = 1$ or $2qr = \frac{1}{16}$ or $\frac{25}{64} + 2"\frac{5}{8}"q + 2"\frac{5}{8}"r + q^2 + \frac{1}{16} + r^2 = 1$	B1
	Any two equations from above	B1
	$\frac{3}{8}q - q^2 = \frac{1}{32}$	dM1
	$q = \frac{1}{4}$	A1
	$\mathrm{P}(M = 50) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$ *	A1cso*
		(7)
		Total 10

Notes

(a)	**B1**	Any one of the given reasons. Ignore extraneous non-contradictory reasons.
(b)	**B1**	Idea of list(oe). Need all (oe) (eg complete list) and members.
(c)	**B1**	The members/a member
(d)	**M1**	Correct method, may be implied
	A1	$p = \frac{5}{8}$ or $\mathrm{P}(X = 20) = \frac{5}{8}$
	B1	One equation in q and r from use of $p + q + r = 1$, $\mathrm{P}(M = 60)$ or $\sum \mathrm{P}(M=m) = 1$ see Note (allow ft on their value of p)
	B1	Two correct equations in q and r Some will substitute directly into the third equation so may see: $\frac{25}{64} + \frac{5}{4}q + \frac{5}{128q} + q^2 + \frac{1}{16} + \frac{1}{1024q^2} = 1$ which is correct and scores B1B1
	dM1	(dep on 1st B1) Correct method to solve simultaneous equation leading to a probability for q or r (may be implied by $q = \frac{1}{4}$ or $r = \frac{1}{8}$ provided B1B1 scored)
	A1	Correct probability for q (dependent on all previous marks in part (d))
	A1cso*	Correct solution with use of $\mathrm{P}(M = 50) = q^2$ and all previous marks awarded.
	Note:	

m	20	35	45	50	60	70
$\mathrm{P}(M=m)$	$\frac{25}{64}$	$2pq$	$2pr$	q^2	$\frac{1}{16}$	r^2

$$\frac{25}{64} + 2pq + 2pr + q^2 + \frac{1}{16} + r^2 = 1$$